

$$\text{So, } \frac{1}{r} \frac{d}{dt} \left( r^2 \frac{d\theta}{dt} \right) = 0$$

$$\text{So, } r^2 \frac{d\theta}{dt} = K = \text{Constant}$$

This is the Kepler's law of area.

Hence, the angular momentum  $p_\theta$  is a constant as given by

$$p_\theta = m r^2 \frac{d\theta}{dt} \quad (m = \text{mass of the electron})$$

Thus from Sommerfeld's quantum restriction

$$\begin{aligned} hR &= \oint p_\theta d\theta = p_\theta \oint d\theta \\ &= m r^2 \frac{d\theta}{dt} \times 2\pi \\ &= p_\theta \times 2\pi \end{aligned}$$

$$p_\theta = k \left( \frac{hR}{2\pi} \right)$$

This is in good agreement with postulate of Bohr according to which the angular momentum of an electron in any stationary orbit is an integral multiple of  $\frac{h}{2\pi}$ .

Proof for  $b/a = k/w$

Momentum along the radius =  $p_r = m \cdot \frac{dr}{dt}$

$$\therefore p_r dr = m \cdot \frac{dr}{dt} dr$$

$$= m \left( \frac{dr}{d\theta} \frac{d\theta}{dt} \right) \frac{dr}{d\theta} d\theta$$

$$= m \left( \frac{dr}{d\theta} \right)^2 \frac{d\theta}{dt} d\theta$$

$$= \left( \frac{1}{r} \frac{dr}{d\theta} \right)^2 p_\theta \cdot d\theta \rightarrow (I)$$

$$p_\theta = m r^2 \frac{d\theta}{dt}$$