elliptical nature of the orbit ⟶ different position of the electron on the locus ⟶ different values of r ⟶ electrostatic attraction varies from to point.

Thus the balancing force (i.e the centrifugal force) which arises due to the angular motion also varies from point to point.

Velocity maximum - closest position → perihelion

velocity minimum → farthest position → aphelion

Due to the variation of velocity at different points the electron suffers from a relativistic variation in the mass of the electron according to the following relation

$$m = m_0 \left(1 - u^2/c^2\right)^{-1/2}$$

As a result, there is a continual mass variation and it leads to generate a perturbation in the equilibrium force between the electron and the nucleus. Because of this fact, the path of the electron is in fact a rosette which is an ellipse whose major axis precesses slowly in the plane of the ellipse about an axis passing through one of its foci at which the nucleus is residing.

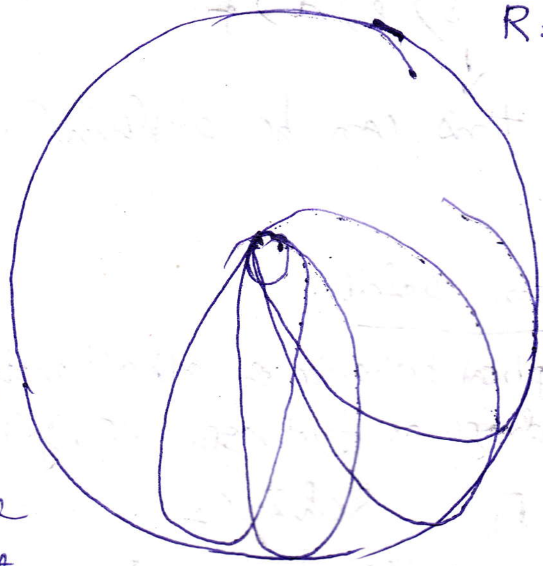
$$E_n = \frac{-mZ^2e^4}{8\varepsilon_0^2h^2n^2}\left[1 + \frac{\alpha^2Z^2}{n}\left(\frac{1}{k} - \frac{3}{4n}\right)\right]$$

$$R = \frac{me^4}{8\varepsilon_0^2ch^3}$$

or $E_n = -\frac{RchZ^2}{n^2} - \frac{Rch\alpha^2Z^4}{n^3}\left(\frac{1}{k} - \frac{3}{4n}\right)$

α is a constant, called fine structure constant. It has got not unit.

$$\alpha = \frac{e^2}{2\varepsilon_0 ch} = \frac{1}{137}$$



Precessing Sommerfeld's electron orbit (rosette)