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WAVE MOTION : A simple example of wave motion in one dimension can be provided by a displacement ~~moving~~ along a string, fixed at one end only. Suppose the string vibrates with a frequency ν producing waves of length λ which travel in the positive x -direction with a velocity u . The amplitude y , i.e., the height of the wave at any point x and time t , that is,

$$y = f(x, t)$$

Such a wave motion in one dimension can be represented mathematically by the general partial differential equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{u^2} \frac{\partial^2 y}{\partial t^2}$$

Standing Waves :

Vibration of a string fixed at its two ends produce standing wave or stationary wave.

With separated variables, the amplitude y of the wave may be expressed as

$$y = f_1(x) f_2(t)$$

$$f_2(t) = A \sin 2\pi \nu t$$

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Differentiating y twice with respect to t at constant x

$$\frac{\partial y}{\partial t} = f_1(x) \cdot 2\pi \nu \cdot A \cos 2\pi \nu t$$

$$\frac{\partial^2 y}{\partial t^2} = -4\pi^2 \nu^2 \underline{f_1(x) A \sin 2\pi \nu t}$$

$$= -4\pi^2 \nu^2 f_1(x) f_2(t) \rightarrow \text{(i)}$$

But we know, $\frac{\partial^2 y}{\partial x^2} = \frac{1}{u^2} \frac{\partial^2 y}{\partial t^2}$

$$\therefore \frac{\partial^2 y}{\partial x^2} = -\frac{4\pi^2 \nu^2}{u^2} f_1(x) f_2(t) \rightarrow \text{(ii)}$$