~~or $f_1(x)$ As~~

Again, $y = f_1(x) f_2(t)$

$$\frac{dy}{dx} = f_2(t) \cdot \frac{df_1(x)}{dx}$$

$$\frac{d^2y}{dx^2} = f_2(t) \cdot \frac{d^2 f_1(x)}{dx^2} \longrightarrow \text{(iii)}$$

Comparing (ii) and (iii) we get

$$f_2(t) \frac{d^2 f_1(x)}{dx^2} = -\frac{4\pi^2 \nu^2}{u^2} f_1(x) f_2(t)$$

or, $$\frac{d^2 f_1(x)}{dx^2} = -\frac{4\pi^2 \nu^2}{u^2} f_1(x) \longrightarrow \text{(iv)}$$

The velocity of the wave, u, is related to the frequency ν, by the relation

$u = \lambda \cdot \nu$, where λ is the corresponding wavelength

So, $\frac{1}{\lambda} = \frac{\nu}{u}$

or, $\frac{1}{\lambda^2} = \frac{\nu^2}{u^2}$

Now, eqn. (iv) takes the form

$$\frac{d^2 f_1(x)}{dx^2} = -\frac{4\pi^2}{\lambda^2} f_1(x) \longrightarrow \text{(v)}$$

Extending this to a wave moving in three dimensions $f_1(x)$ is replaced by a new function depending on the three Cartesian coordinates: $\Psi(x, y, z)$. Writing ~~Ψ~~ simply Ψ for $\Psi(x, y, z)$, equation (v) can be transformed to

$$\frac{d^2\Psi}{dx^2} + \frac{\partial^2\Psi}{dy^2} + \frac{\partial^2\Psi}{dz^2} = -\frac{4\pi^2}{\lambda^2}\Psi$$

or, $$\nabla^2\Psi = -\frac{4\pi^2}{\lambda^2}\Psi \quad \left[\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]$$