

Matter Wave: According to the fundamental postulates of Schrodinger equation (ψ) may be applied to all particles. Thus the equation may be further adjusted by inserting the mass (m) and velocity (v) of the particle concerned from the de Broglie relationship $\lambda = \frac{h}{mv}$.

$$\nabla^2 \psi = - \frac{4\pi^2 m^2 v^2}{h^2} \psi \rightarrow (vi)$$

The total energy E of the particle must be equal to the sum of its potential energy V and kinetic energy $\frac{1}{2}mv^2$.

$$E = V + \frac{1}{2}mv^2 \quad \therefore mv^2 = 2[E - V]$$

So, the equation (vi) can be written as

$$\begin{aligned} \nabla^2 \psi &= - \frac{4\pi^2 m \cdot mv^2}{h^2} \psi \\ &= - \frac{8\pi^2 m (E - V)}{h^2} \psi \end{aligned}$$

$$\therefore \nabla^2 \psi + \frac{8\pi^2 m (E - V)}{h^2} \psi = 0$$

$$\therefore \left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0 \right]$$

This is the wave equation of Schrodinger in the time independent form.

$$\nabla^2 \psi + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0$$

$$\therefore \left(-\frac{h^2}{8\pi^2 m} \nabla^2 + V \right) \psi = E \psi$$

$$\therefore \hat{H} \psi = E \psi$$

\hat{H} is called the Hamiltonian Operator.