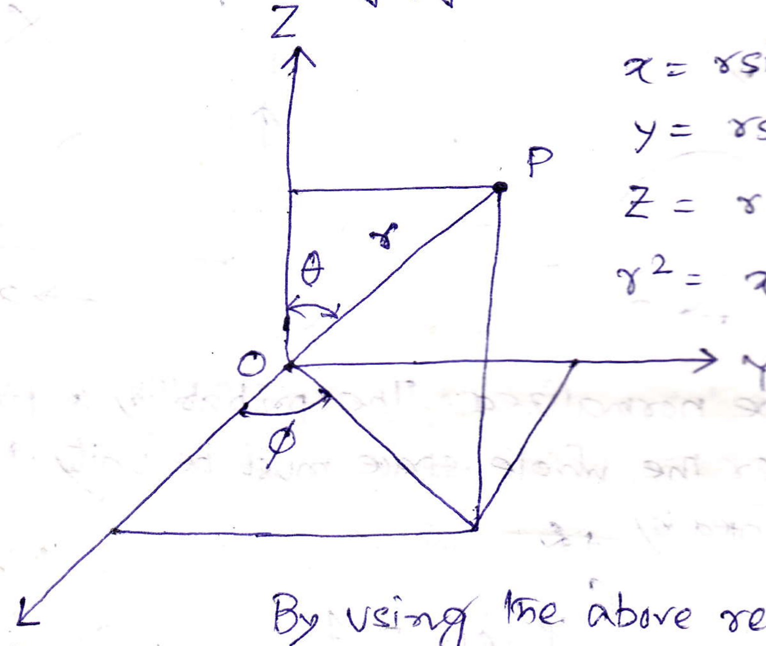
Wave functions for hydrogen like systems :

Schrödinger wave equation for hydrogen like system is given by

$$\nabla^2\psi + \frac{8\pi^2\mu}{h^2}\left(E + \frac{Ze^2}{r}\right)\psi = 0$$

It becomes easier to solve the above equation if it is expressed in terms of spherical polar coordinates. The relationship between the Cartesian coordinates x, y, z and the polar coordinates r, θ and φ for a particular point P is shown in the following figure.



$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$

$r^2 = x^2 + y^2 + z^2$

By using the above relations the wave equation is transformed into spherical polar coordinate as

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{d\psi}{\partial\theta}\right) +$$

$$\frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2} + \frac{8\pi^2 m}{h^2}\left(E + \frac{Ze^2}{r}\right)\psi = 0 \rightarrow (1)$$

The wave function ψ is a function of three variables r, θ and φ. The wave function can be split into three parts, each of which is a function of a single variable.

$$\psi(r,\theta,\phi) = R(r)\,\Theta(\theta)\,\Phi(\phi)$$

$$= R\Theta\Phi$$